gain-bandwidth product curve is less than the calculated values, but there was no way to adjust and optimize the coupling once the loop was formed. In any tunable maser, some method of adjusting the coupling could be easily incorporated to offset this trouble. Fig. 3 shows pump power necessary for saturation vs signal input power. The amount of pump power necessary for saturation is larger than that usually required for the resonant cavity case, but not significantly so. We also noted that the gain of the maser is not dependent upon the frequency stability of the pump source. It was also noted that oscillations could be started over nearly the entire linewidth of the ruby, *i.e.*, 50-Mc tuning range of the pump frequency with little variation in the pump power.

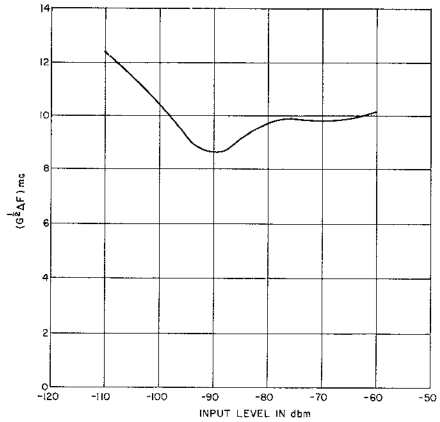

Fig. 2—Gain bandwidth product vs input level in dbm.

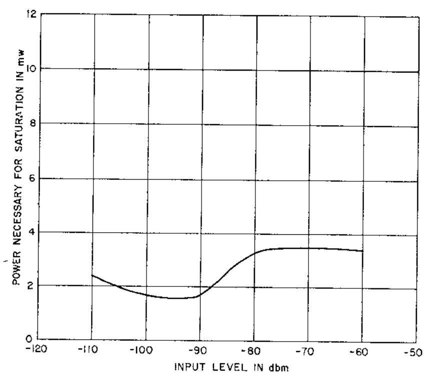

Fig. 3—Saturation pump power vs signal input level in dbm.

In conclusion, it appears feasible that a tunable cavity maser at S band could be developed without recourse to a dual-mode cavity system. This would greatly enhance the practicability of this type of amplifier for the 2000 Mc region where masers can be used efficiently for telemetry purposes.

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Higher-Order Evaluation of Dipole Moments of a Small Circular Disk for Arbitrary Incident Fields*

In a recent note[1] the induced electric and magnetic dipole moments P and M due to the diffraction of a plane wave on a small circular disk were given. The expression for the electric dipole moment holds, however, only if the electric field vector is parallel to the plane of incidence. In a more general approach the case for an arbitrary primary field has been examined, and the following expressions have been obtained:

$$P_x = \frac{16}{3} a^3\epsilon_0 \left[E_x{}^i + \frac{(ka)^2}{30}\left(13E_x{}^i - \frac{3}{k^2}\frac{\partial^2 E_x{}^i}{\partial z^2} + \frac{2j}{\omega\epsilon_0}\frac{\partial H_z{}^i}{\partial y}\right) - j\frac{8}{9\pi}(ka)^3 E_x{}^i + 0((ka)^4)\right],$$

$$P_y = \frac{16}{3} a^3\epsilon_0 \left[E_y{}^i + \frac{(ka)^2}{30}\left(13E_y{}^i - \frac{3}{k^2}\frac{\partial^2 E_y{}^i}{\partial z^2} - \frac{2j}{\omega\epsilon_0}\frac{\partial H_z{}^i}{\partial x}\right) - j\frac{8}{9\pi}(ka)^3 E_y{}^i + 0((ka)^4)\right],$$

$$M_z = -\frac{8}{3} a^3 \left[H_z{}^i - \frac{(ka)^2}{10}\left(3H_z{}^i + \frac{1}{k^2}\frac{\partial^2 H_z}{\partial z^2}\right) + j\frac{4}{9\pi}(ka)^3 H_z{}^i + 0((ka)^4)\right].$$

The axis of the disk is along the z direction. The incident fields are evaluated at the center of the disk.

We now consider a plane wave ($\boldsymbol{E}^i$, $\boldsymbol{H}^i$) incident in the xz plane and with an angle of incidence θ. Using the expressions above we obtain

$$P_x = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{10}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_x{}^i,$$

$$P_y = \frac{16}{3} a^3 \left[1 + \left(\frac{8}{15} - \frac{1}{6}\sin^2\theta\right)(ka)^2 - j\frac{8}{9\pi}(ka)^3 + \cdots \right] E_y{}^i,$$

$$M_z = -\frac{8}{3} a^3 \left[1 - \frac{1}{10}(2 + \sin^2\theta)(ka)^2 + j\frac{4}{9\pi}(ka)^3 + \cdots \right] H_z{}^i.$$

P_x and M_z agree with the values given in reference [1].

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[1] W. H. Eggimann, "Higher order evaluation of dipole moments of a small circular disk," IRE Trans. on Microwave Theory and Techniques, vol. 8, p. 573; September, 1960.

Capacitance Definitions for Parametric Operation*

The purpose of this note is to reconcile the various definitions of nonlinear or equivalent time-varying capacitance which appear in the literature concerned with parametric devices. The problem considered is one of a nonlinear reactive element, let us say a capacitance, which is pumped by strong pump source at frequency f_p and which couples two circuit modes at frequencies f_s and f_i, usually called the signal and idling frequencies. For parametric operation we demand either $f_p = f_i + f_s$ or $f_p = f_i - f_s$. The nonlinear element (*i.e.*, a capacitance) has a charge-voltage characteristic given by

$$q = f(V). \tag{1}$$

Let us imagine that now we have applied a strong pump voltage V_p together with a dc bias voltage V_0 and subsequently we shall be concerned with the behavior upon application of small signal voltage δv at signal and idling frequencies. We can now define several capacitances:

1) The total capacitance C_T is defined as the ratio of total charge to total voltage, or

$$q = C_T V. \tag{2}$$

Obviously from (1)

$$C_T = \frac{f(V)}{V}; \tag{3}$$

this is the definition of capacitance used by Heffner and Wade.[1]

2) The incremental capacitance C_i is defined by

$$\delta q = C_i \delta V. \tag{4}$$

This is the definition used by Rowe.[2]

Two questions arise: first, in definition 1, what is the relationship between the time-varying capacitance produced by the pump alone to that which is effective in producing parametric action; second, what is the relationship between the two definitions of capacitance?

The first question already has been answered,[1] but it is perhaps worthwhile to add a few more details. If the biased capacitance is acted on by the pump voltage alone, then we would measure a charge,

$$q = (C_{T0} + C_{TP})(V_0 + V_p), \tag{5}$$

where we consider the capacitance to have a dc part C_{T0} and a time varying part C_{TP} varying sinusoidally at the pump frequency (for simplicity we will neglect harmonic terms). The time varying part could be measured or could be inferred from a static plot of the nonlinear characteristic. We necessarily demand that the amplitude of the time-varying part be less than or at most equal to the dc part in order to have the

* Received by the PGMTT, October 4, 1960.

[1] H. Heffner and G. Wade, *J. Appl. Phys.*, vol. 29, pp. 1321–1331; September, 1958.

[2] H. E. Rowe, "Some general properties of nonlinear elements—II. Small signal theory," Proc. IRE, vol. 46, pp. 850–860; May, 1958.